The quickest way to break a safe and other graph theory problems

Summer Camp of School 57. August 2013

1st Lesson

Graph is a pair G = (V, E), where V = {v_1, …, v_n} is the vertex set, and E is the set (or multiset) of pairs of vertices, i. e. E is subset of V×V.

Graph V is called **simple graph** if it has no loops and no multiple edges. Usually we will deal with simple graphs.

Two graphs are called **isomorphic** if one can rearrange the vertices of second graph so that structures of the graphs are the same. (You can define the graph isomorphism more carefully using bijection conception if you like.)

1. Draw all non-isomorphic graphs with 4 or less vertices. How many such graphs exist?

2. Let G_n be the simple graph with V={v_1, …, v_n}, and pair (v_i, v_j) is connected with an edge if and only if i and j are relatively prime (i. e. there are no numbers m greater than 1 such that m | i and m | j). Draw the graphs G_6 and G_8.

3. How many graphs (maybe isomorphic) with vertex set {1, 2, …, 57} exist? How many of them have 2013 edges?

4. There are 10 students in the class. Can each of them have different number of friends? (If Alice is a friend of Bob, then Bob is a friend of Alice.)

Graph on n vertices is called **complete** if it has n(n-1)/2 edges, and is called **empty** if it has no edges.

The **clique number** ω(G) is the biggest integer ω wuch that G has a complete subgraph on ω vertices.

The **independence number** α(G) is the biggest integer α wuch that G has an empty subgraph on α vertices.

5. Find the clique and independence numbers of graphs from Problem 2.

6. Draw a graph with |V(G)| > α(G) + ω(G). Does there exist a graph with |V(G)| < α(G) + ω(G) – 1 ?

8. There are 200 cities in Indonesia, and each city has flights to at least 100 cities. Prove that if we cancel any 99 flights, it will still possible to get from any city to any other city (maybe with flight changes).

9. Graph G has 12 vertices, and each its subgraph on 9 vertices has induced subgraph K_5. Prove that G has induced subgraph K_6.

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2^{nd} Lesson

11. How many non-isomorphic graphs with 8 vertices and 25 edges exist?

12. How many independent sets of size n exist in $K_{k,l}$?

13. Prove that the number of non-intersecting by edges Hamiltonian cycles in K_n is [(n-1)/2].

14. How many different Hamiltonian cycles are in K_n? How many – in $K_{k,l}$?

15. Build the De Bruijn graph and find the binary De Bruijn sequence of order 4

a) starting from 1011, b) finishing with 1010.

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